

4.4 – Linear Independence

Definition: If $S = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_r\}$ is a set of two or more vectors in a vector space V , then S is said to be a **linearly independent set** if no vector in S can be expressed as a linear combination of the others. A set that is not linearly independent is said to be **linearly dependent**. If S has only one vector, we will agree that it is linearly independent if and only if that vector is nonzero.

Theorem 4.4.1 A nonempty set $S = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_r\}$ in a vector space V is linearly independent if and only if the only coefficients satisfying the vector equation $k_1 \mathbf{v}_1 + k_2 \mathbf{v}_2 + \dots + k_r \mathbf{v}_r = \mathbf{0}$ are $k_1 = 0, k_2 = 0, \dots, k_r = 0$.



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- a. Show that the three vectors $\mathbf{v}_1 = (0, 3, 1, -1)$, $\mathbf{v}_2 = (6, 0, 5, 1)$, $\mathbf{v}_3 = (4, -7, 1, 3)$ form a linearly dependent set in R^4 .
- b. Express each vector in part (a) as a linear combination of the other two.

#14 In each part, let $T_A : R^3 \rightarrow R^3$ be multiplication by A and let $\mathbf{u}_1 = (1, 0, 0)$, $\mathbf{u}_2 = (2, -1, 1)$, and $\mathbf{u}_3 = (0, 1, 1)$. Determine whether the set $\{T_A(\mathbf{u}_1), T_A(\mathbf{u}_2), T_A(\mathbf{u}_3)\}$ is linearly independent in R^3 .

a. $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & -3 \\ 2 & 2 & 0 \end{bmatrix}$

b. $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -3 \\ 2 & 2 & 0 \end{bmatrix}$

